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DISPERSION OF EFFLUENT PLUMES FROM DIFFUSERS ON NEAR-SHORE REGIONS OF THE GREAT LAKES

VOLUME I

INITIAL MIXING PROCESSES

AN ASSESSMENT PROCEDURE

PREPARED FOR

WORKING GROUP II
OF THE WATER MANAGEMENT
STEERING COMMITTEE

Ministry
of the
Environment

Ontario

The Honourable
Jim Bradley
Minister

Rod McLeod
Deputy Minister

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Vol. 1

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An Assessment Procedure
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of the Water Management
Steering Committee

Y. Hamdy
Great Lakes Section
Water Resources Branch
August, 1981

NOTE

The receiving water assessment techniques presented in this report describe the methods commonly in use within the Ministry. Alternative techniques exist and/or will eventually be developed. Consultation with Ministry of the Environment staff is advisable to determine the suitability of the alternative techniques.

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INTRODUCTION

In managing the wastewaters (municipal, industrial and heat) different methods of treatment and disposal are used depending on economic, engineering and environmental quality considerations. The discharge of treated wastewaters into a natural body of water is a viable economic and engineering solution.

The Ontario Ministry of the Environment has established water quality objectives to regulate the adverse effects of such discharges on the receiving waters (29). A policy and set of requirements in the Ministry's water management program are designed to keep mixing zones associated with these discharges as small as possible while ensuring that the Provincial Water Quality Objectives are met at the boundary of the mixing zone (29).

Generally, a properly designed submerged multiport diffuser outfall should result in high initial dilutions. A clear understanding of physical processes associated with diffuser outfall discharges is needed so that an accurate estimate of initial dilutions and a reliable assessment of the impact of wastewater discharges on the environment can be made.

In the past, engineers used dilution curves established by Rawn et al (30) to estimate initial dilution ratios for diffuser outfall discharges into the Ontario Coastal Zone of the Great Lakes. These curves were developed for discharges into deep sea waters (relative density difference between effluent and sea water is about 0.025). Thus the use of the curves for diffusers in the Great Lakes which are characterized by shallow depths and low relative density differences ($\approx$.004) would fail to give correct predictions.

This paper reviews the buoyant jet theory in "deep" and "shallow" receiving waters. Analyses of predictive models and their applicability to submerged diffuser outfall discharges into the coastal zone of the Great Lakes are presented. The review is focused on the predictions of initial dilution factors under stagnant or low current conditions since they provide a worst case minimum dilution that is improved by the presence of moderate-to-strong longshore currents. Simplified methods are summarized and nomograms constructed to estimate initial dilution factors for discharges from different types of diffuser outfalls.

JET TYPES

Characteristics of jets issuing from round or slot ports are governed by: depth of submergence (y), diameter (D) or width (B) of the port, density difference between the jet and the ambient receiving water and the jet velocity (U_o).

The Froude number is a convenient dimensionless parameter describing the ratio of the inertia forces to the buoyancy (gravitational) forces, which can be expressed as:

$$F = U_o/(g'\ D)^{1/2} \quad \text{for a round/(circular) port} \quad \ldots\ldots\ldots\ldots\ldots \quad (1a)$$

$$ = U_o/(g'\ B)^{1/2} \quad \text{for a slot/(rectangular) port} \quad \ldots\ldots\ldots\ldots \quad (1b)$$

where $g' = g(\rho_a - \rho_e)/\rho_a$; g is acceleration due to gravity, ρ_a is the ambient density, ρ_e is the effluent density.

Near the outlet inertia forces are large enough to negate the effect of buoyancy while at greater distances from the source the buoyancy forces are more important than the inertia (momentum) forces (1).

Depending on the Froude number, we may have 3 types of jets:

1) Simple Jet: A non-buoyancy case where the jet momentum is predominant. Usually called a momentum source. The Froude number is large ($F \rightarrow \infty$).

2) Simple Plume: A buoyancy case where the initial momentum is negligible. Usually called a buoyancy source. The Froude number is small ($F \rightarrow 0$).

3) Forced Plume: An intermediate case, where both buoyancy and momentum are important. The Froude number has a finite value ($0 < F < \infty$).

Figure 1 illustrates the jet trajectory for each case. It can be seen that the jet trajectory for a simple jet is mainly horizontal (parallel to the lake bottom); whereas the trajectory for a simple plume will be vertical. For an intermediate case (forced plume) the jet is no longer a straight, but rather a curved jet trajectory.

These types of jets can be found in diffuser outfall discharges. For example, diffuser outfalls with ports discharging in the same direction with high velocity (i.e. high Froude numbers), usually produce a momentum source. These outfalls are called staged or unidirectional diffusers (Figure 2). If the discharge velocity is not high, so that both velocity and density difference (between the effluent and ambient receiving water) are important, we will have an intermediate case. If a diffuser outfall is equipped with ports discharging in opposite directions (i.e. alternating diffuser, Figure 2), the net horizontal momentum will be negligible after an initial mixing zone and only buoyancy will be important.

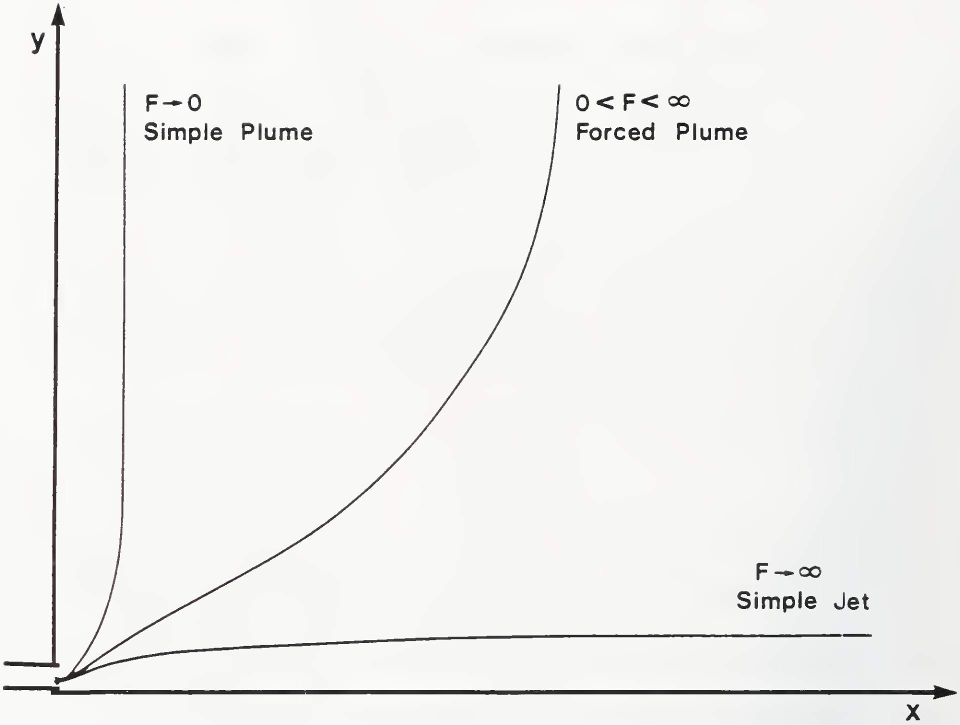


FIGURE 1 TRAJECTORIES FOR DIFFERENT TYPES OF JETS

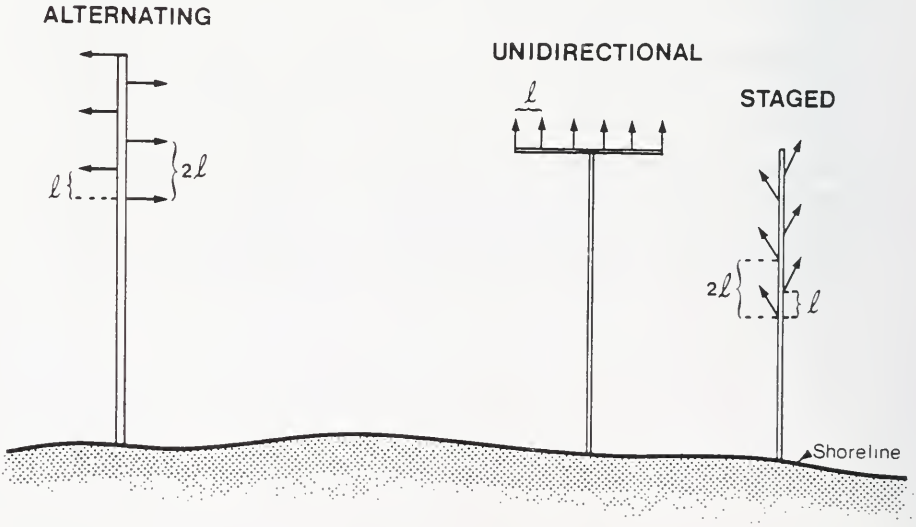


FIGURE 2 DIFFUSER TYPES

MIXING ZONES

In any type of jet, the mixing process can be divided into four essential mixing zones (Figure 3) as indicated by Abraham (1, 2) and Hart (15):

(I) Zone of Flow Establishment:

In this zone the discharge plume consists of a core of uniform concentrations C_0 and a surrounding boundary layer of concentrations C less than C_0. Albertson et al (6) found that in a case of a round port, the core extends horizontally to a distance equal to 6 times the jet diameter; and for a rectangular port (slot port), the core extends to a distance of 5 times the port width.

(II) Zone of Established Flow:

In this zone the concentration decreases while the jet width increases until it impinges on the water surface.

(III) Surface Impingement Zone:

Upon impingement, the jet changes direction as it spreads laterally creating a zone of essentially the same concentration developed in the zone of established flow. This zone prevents further dilution of effluent.

(IV) Horizontal Surface Flow:

In this zone, mixing will be affected by lake currents as it spreads out.

BOUNDARY EFFECTS

The single most important variable which affects the behaviour of submerged discharges and hence the applicability of predictive

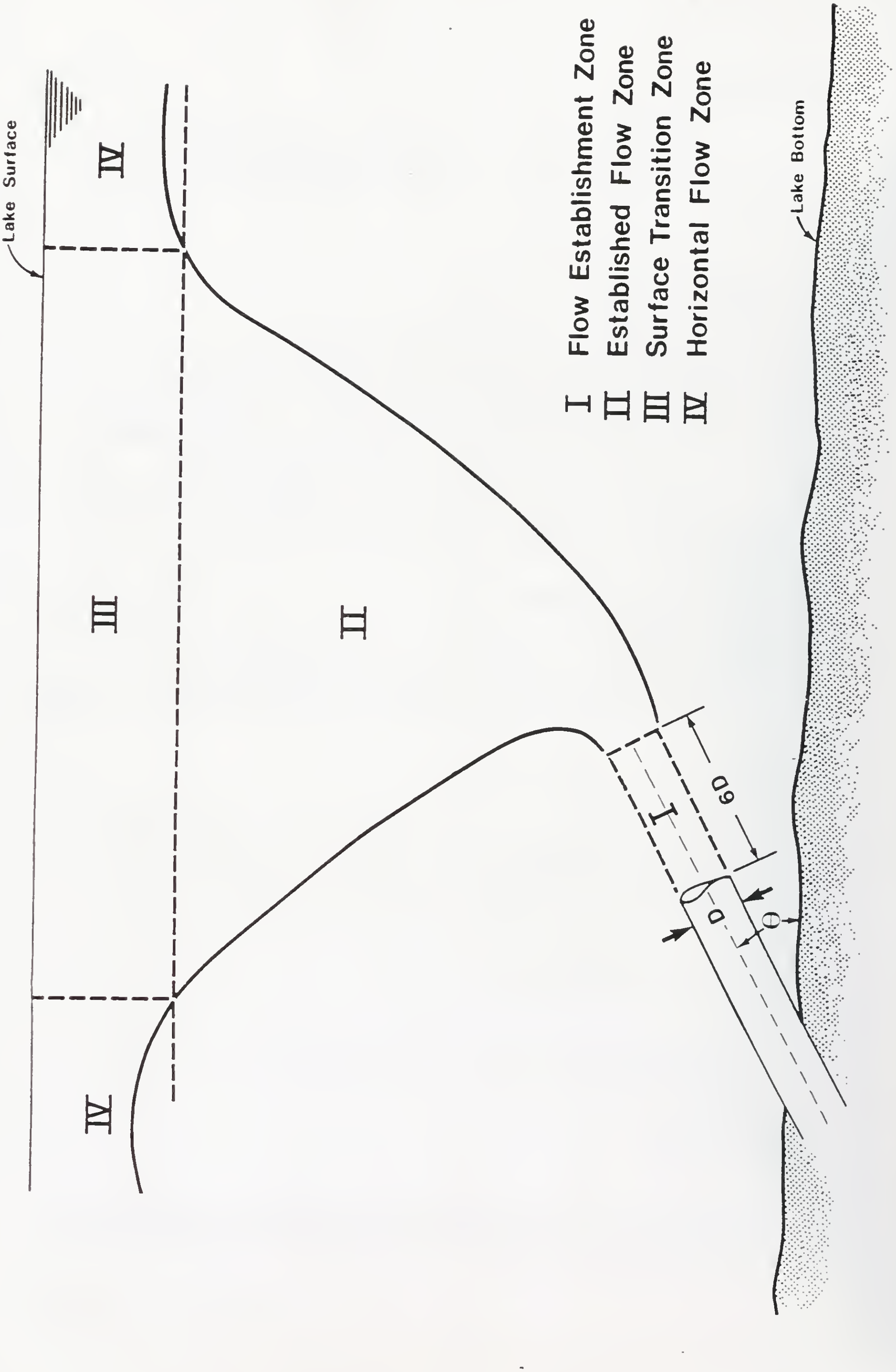


FIGURE 3 MIXING ZONES

models is the water depth. Submerged discharges into deep receiving waters generally exhibit stable conditions that is, upon impingement on the free surface, the discharge spreads laterally in the form of a stable density current. These stable conditions which are usually developed for low discharge rates, create a stratified near-field flow regime which allows fresh ambient waters to entrain with the jet flow (Figure 4).

The vertical distance below the water surface up to which effective entrainment occurs is dependent on the thickness of the surface impingement zone. The thickness of this zone may vary from 8% to 20% of the total water depth (2, 18, 22).

In shallow receiving waters, entrainment into the jet is limited and the jet impingement at the water surface usually results in a complete vertical mixing and an unstratified near-field flow regime is established which is called unstable near-field (Figure 4). This type of near-field flow regime is usually developed under high discharge rates (i.e. high Froude numbers).

The differentiation between "deep" and "shallow" receiving water is therefore dependent on the actual depth and the dynamic and geometric characteristics of the discharge. These factors control the stability of the discharge.

STABILITY ANALYSIS

The governing parameters for the near-field stability are Froude number (F), water depth to port diameter ratio (H/D) and port spacing to port diameter ratio (ℓ/D). The definition of port spacing is illustrated in Figure 2.

Stability criteria for multiple port diffusers where jet interference occurs were developed according to the diffuser type (8, 18, 23).

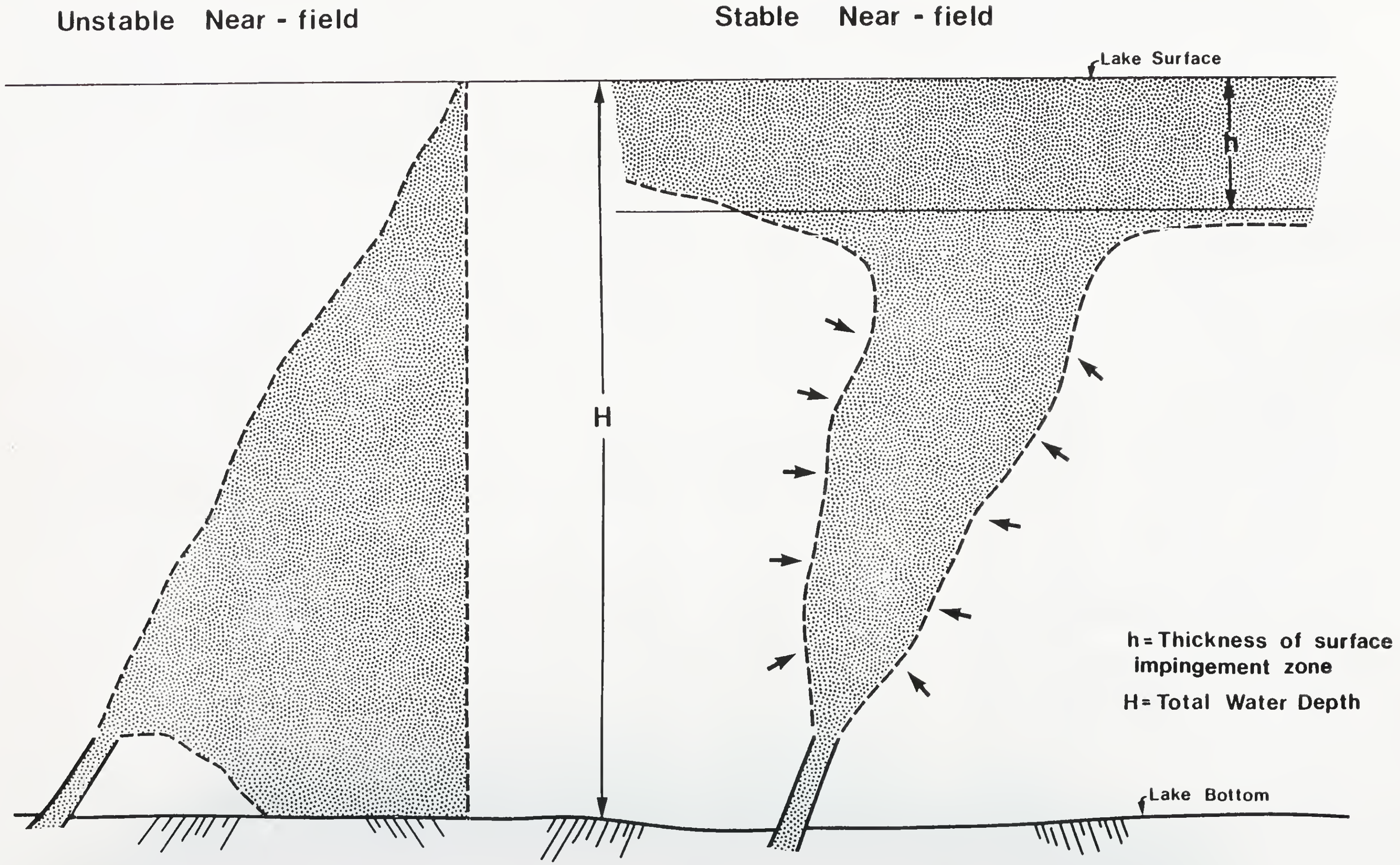


FIGURE 4 NEAR-FIELD FLOW STABILITY

For alternating diffusers, (no net horizontal momentum) the near-field flow is stable if:

$$H/D > 1.7\ F^{4/3}\ (\ell/D)^{-1/3} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (3)$$

For staged or unidirectional diffusers (negligible buoyancy), the near-field flow is stable when:

$$H/D > 0.46\ F^{4/3}\ (\ell/D)^{-1/3} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (4)$$

Equations 3 and 4 are shown graphically in Figures 5 and 6. Stable near-field flows are found for a diffuser outfall if its (F, H/D) plotted point is situated above its respective ℓ/D line. For example, if F = 20, H/D = 20 for an alternating diffuser (Figure 5), the near-field flow regime will be stable when ℓ/D is greater than 100 and unstable when ℓ/D is smaller than 100.

By rearranging equations 3 and 4, the critical discharge Froude numbers, (F_{cr}) above which the near-field flow is unstable, are:

$$F_{cr} = 0.67(H/D)^{3/4}\ (\ell/D)^{1/4} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (5)$$

for alternating diffusers; and

$$F_{cr} = 1.79(H/D)^{3/4}\ (\ell/D)^{1/4} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (6)$$

for staged and unidirectional diffusers.

It can be readily seen that stable near-field flow regimes can be generated through the use of staged or unidirectional diffusers for a range of Froude numbers wider than those for alternating diffusers.

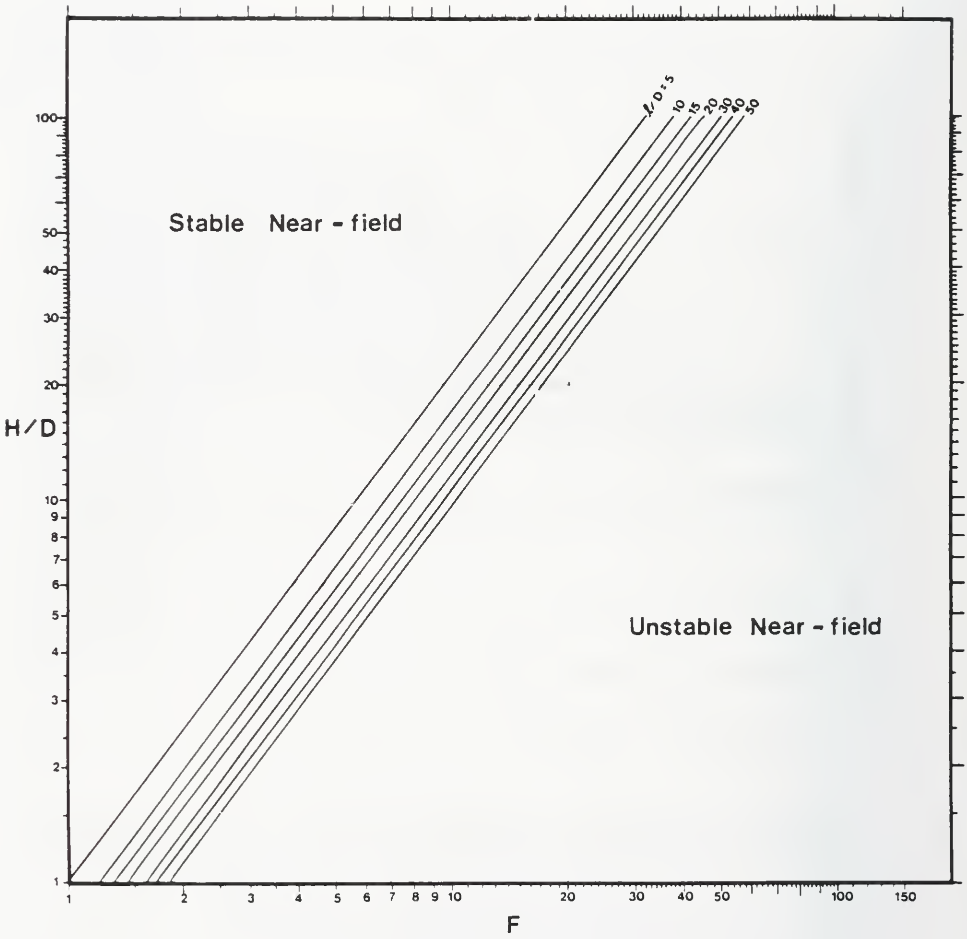


FIGURE 5 STABILITY OF NEAR-FIELD FLOW REGIME FOR ALTERNATING DIFFUSERS- NO NET HORIZONTAL MOMENTUM

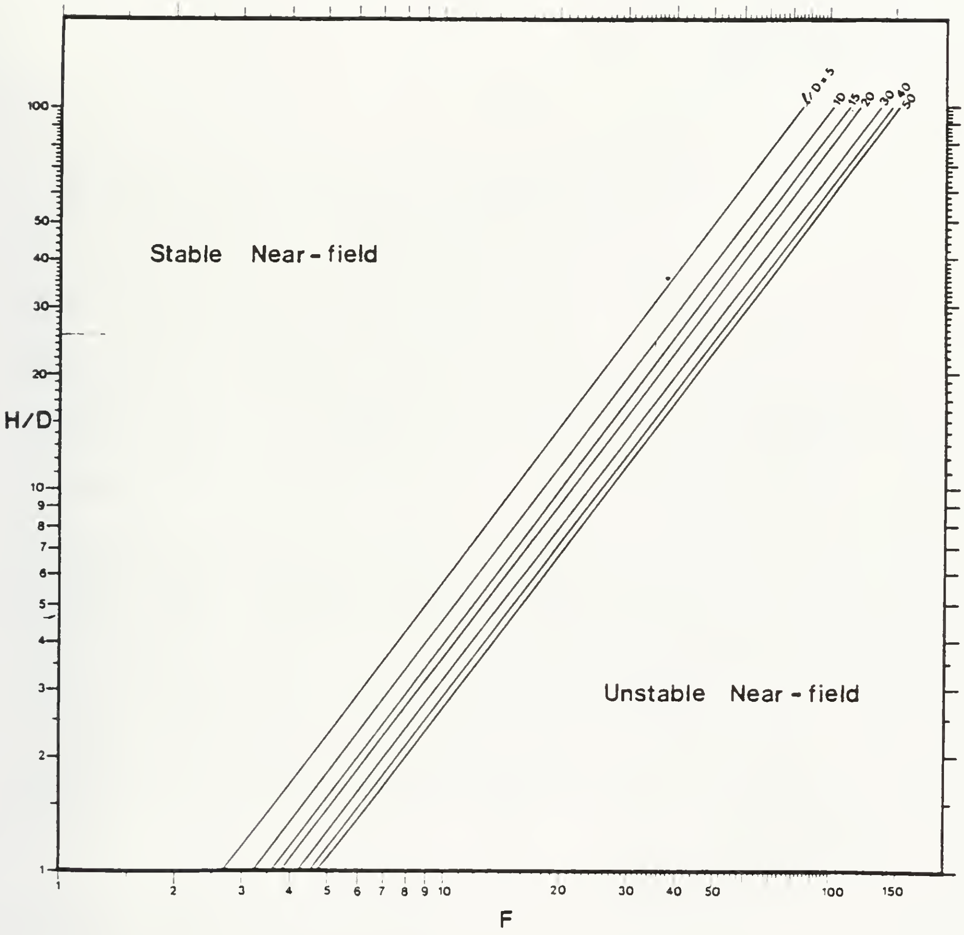


FIGURE 6 STABILITY OF NEAR-FIELD FLOW REGIME FOR STAGED OR UNIDIRECTIONAL DIFFUSERS -NEGLIGIBLE BUOYANCY

In the case of no jet interference where each individual jet maintains its three-dimensional configuration the stability criteria as found by Lee et al (22):

$H/D > 0.23\ F$; and .... (7)

$F_{cr} = 4.4\ H/D$ .... (8)

provided that the relative water depth (H/D) is greater than 6.

The above criteria are used regardless the jet angle (8, 10).

The transition from a point in the stable region to one in the unstable region is continuous in nature as shown in Figure 7 (18, 22). As a jet with low Froude number impinges on the free surface, a thin surface layer is formed followed by a jet-like horizontal spreading accompanied by a weak internal hydraulic jump. As the Froude number increases, the thickness of the layer increases and a weak instability is observed in the near-field. This is characterized by a depth of a hydraulic jump equivalent to one-half of the total depth. For sufficiently high Froude number, the depth of the hydraulic jump will be equal to the total water depth and re-entrainment of previously mixed wastes into the jet region will occur. From previous studies (17, 22), one may suggest that a hydraulic jump of a depth equal to one-half of the water depth occurs at a Froude number equivalent to 30% of the critical value.

DEEP WATER MODELS

All deep water models are based on an integral type analysis the principals of which are documented by Abraham (2), Fan (13), Cederwall (11), Fan and Brooks (14) and Koh and Fan (20) just to a name a few.

The major assumption in deep water models is the similarity of velocity and temperature deficiency (or concentration) profiles. Usually, both are assumed to be Gaussian as shown in Figure 8.

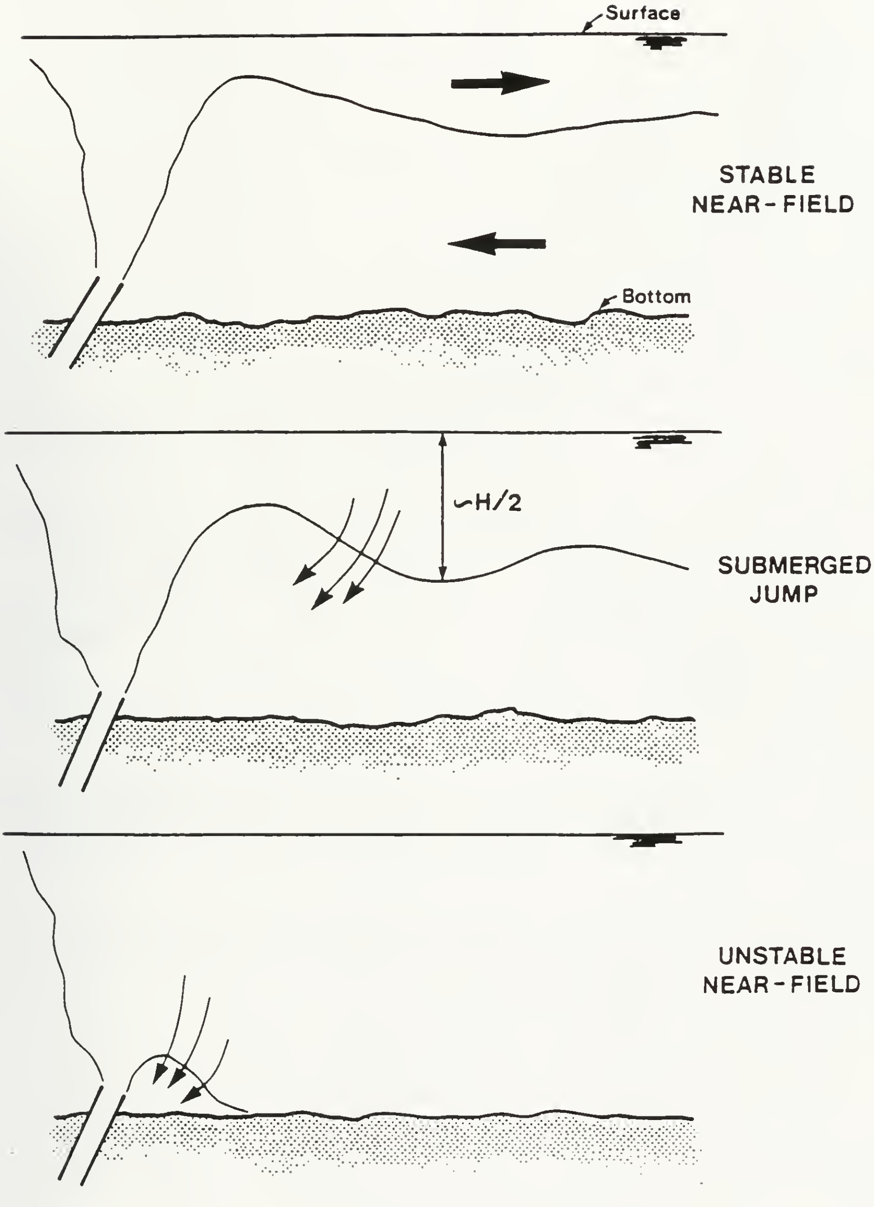
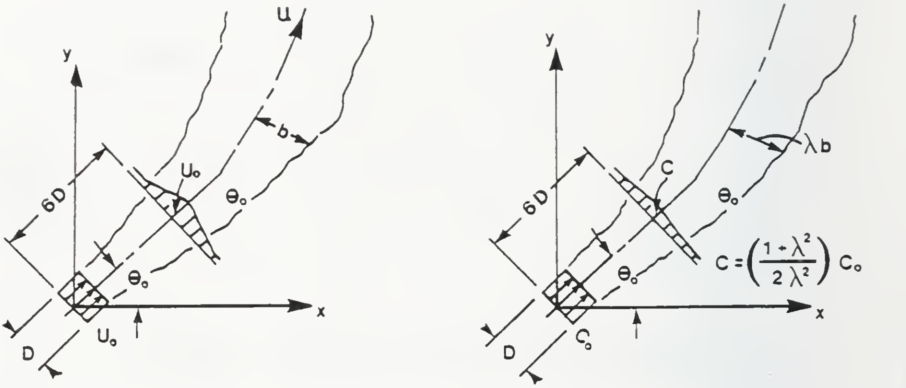


FIGURE 7 TRANSITION FROM STABLE TO UNSTABLE NEAR-FIELD



VELOCITY

CONCENTRATION (DENSITY)

FIGURE 8 VELOCITY AND CONCENTRATION (DENSITY) PROFILES (AFTER FAN AND BROOKS, 1969)

Over the zone of flow establishment, which extends 6 times the port diameter, the distribution of temperature and velocity develop from the "top hat" profiles at the source to Gaussian profiles at the end of the zone. The temperature profile, however, is flatter than the velocity profile. The degree of the relative spread of temperature as compared to the velocity profile depends on the magnitude of the spreading coefficient λ (ratio of turbulent diffusion of mass to that of momentum). The nominal half width of temperature or concentration profile is therefore expressed as λb where b is the nominal half width of the velocity profile. This width (b) is governed by the entrainment process which is the inclusion of ambient water into the jet.

Morton et al (27) proposed that the entrainment process is a function of an entrainment coefficient (α), entrainment velocity (U_e) and the axial jet velocity (U). This function can be expressed as:

$$U_e = \alpha U \quad \text{...} \quad (9)$$

Values of the spreading coefficient (λ) and entrainment coefficient (α) that were determined by many investigators and complied by Abraham (2) for each jet type are shown below.

TABLE 1 - Values of Entrainment and Spreading Coefficients

Type of Jet	Entrainment Coefficient (α)	Spreading Coefficient (λ)
Momentum only (no buoyancy, Simple Jet)	.057 (round port) .069 (slot port)	1.12 (round port) 1.41 (slot port)
Buoyancy only (no initial momentum, Simple Plume)	.082 (round port) .16 (slot port)	1.16 (round port) 1.24 (slot port)

In a submerged multiport diffuser, the round buoyant jets gradually begin to interfere or interact with each other at a certain distance away from the diffuser. In the zone of interaction the typical similarity profiles of the series of round jets are modified to two-dimensional jet profiles and the discharge will behave like a slot buoyant jet. Entrainment and spreading coefficients for round ports are therefore applicable in the region before the jet interference and coefficient for slot ports are used after the interference. The question of which coefficients to use in each region arises.

An examination of a comparison made by Jirka and Harleman (18) between theoretical predictions and experimental data showed that coefficients based on the simple plume theory may be applicable for a discharge densimetric Froude number of 40 or less. For Froude numbers of 40 or greater coefficients can be expressed as a function of the jet local Froude number (17, 18).

Abraham and Jirka (3), however, suggested that for Engineering applications one may be content with a buoyant jet analysis based on constant coefficients to describe entrainment or jet spreading.

Since the assumption of self-similarity is not valid in the zone of transition from round three-dimensional jets to slot two-dimensional plumes, simplifications have been proposed by Abraham (2), and Cederwall (11) based on the equivalent slot concept which states that the dilutions of a multiport discharge after interference are equal to those of an "equivalent slot" discharge. Koh and Fan (20) also assumed that the rate of entrainment into the round jets was equal to that of an equivalent slot jet.

Based on the above criteria, deep water models (14, 20) are applied to determine the relative water depths $(y/D)_t$ at which the interference occurs. These depths are depicted graphically for horizontal and inclined discharges in Figures 9 and 10, respectively for discharge Froude numbers ranging from 1 to 600 and port spacing

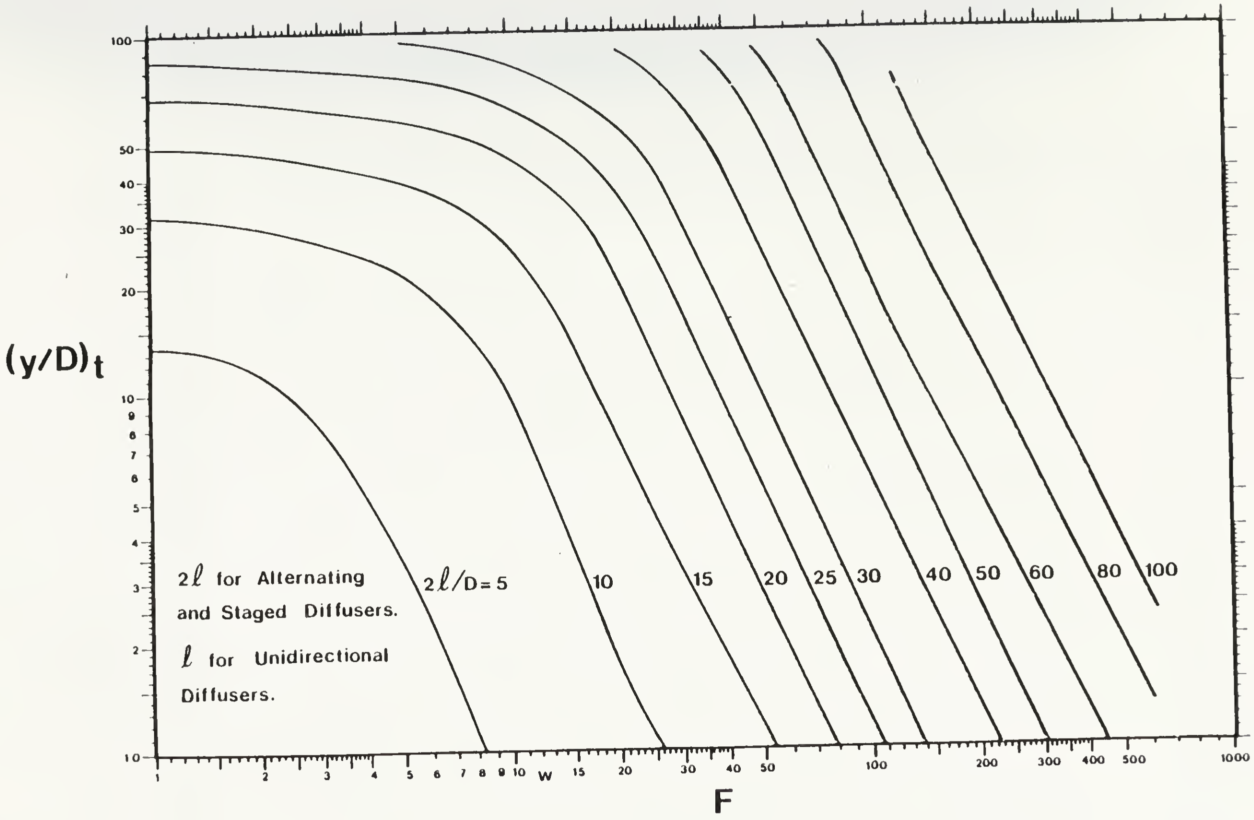


FIGURE 9 JET INTERFERENCE LOCATION $\theta_o = 0°$

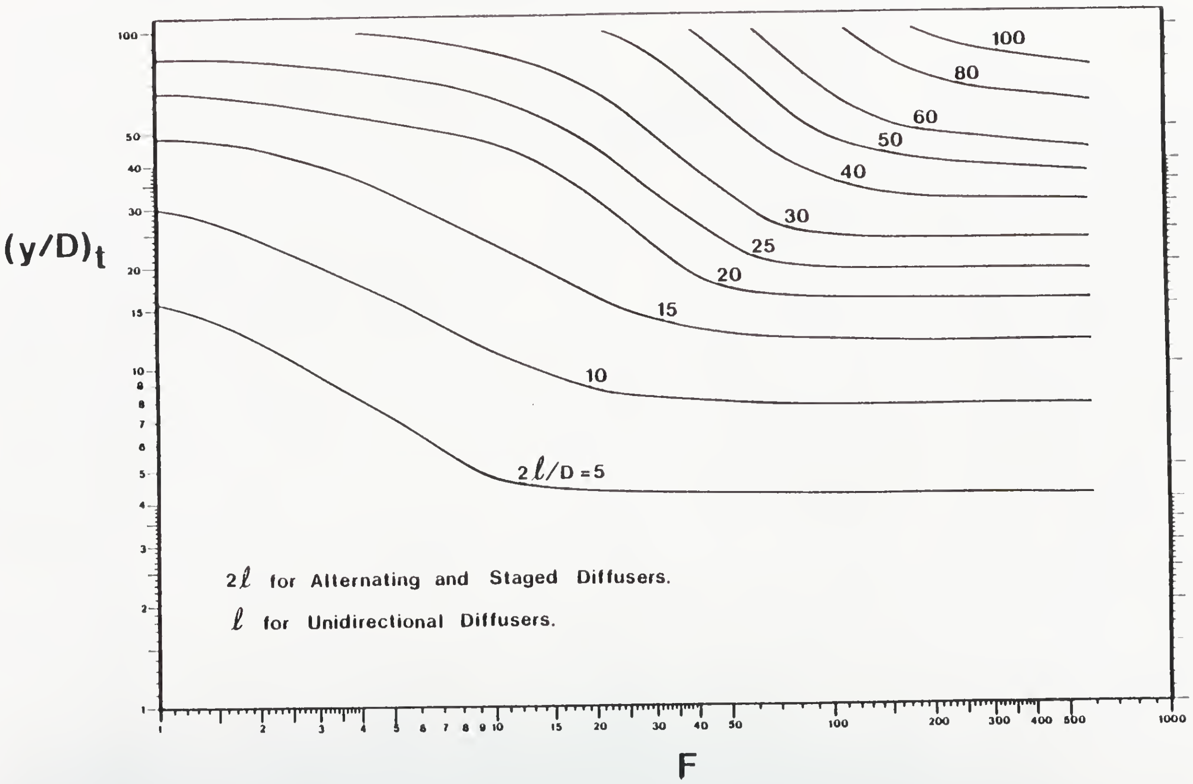


FIGURE 10 JET INTERFERENCE LOCATION $\theta_o = 20°$

diameter ratios from 5 to 100. In the application of deep water models, the spacing of ports on one side of the diffuser must be used for diffusers with staggered ports. From Figure 9 or 10, jet interference occurs if the determined value of $(y/D)_t$ is equal to or less than 0.8 H/D (accounting for the thickness of the surface impingement zone).

For example, the discharge Froude number at which the relative water depth for jet interference $(y/D)_t = 0.8 H/D = 16$ for $2 \ell/D = 40$ is 65 for a jet angle = 0. Based on the preceding stability criteria, unstable discharges will occur at a Froude number $(F_{cr}) = 36$ for staged diffusers and at $F_{cr} = 13$ for alternating diffusers. These Froude numbers are less than those required for jet interaction. Therefore, upon interference, based on deep water models, the discharges are already unstable which may result in interaction earlier than that predicted by 'deep' models. Furthermore, the discharge Froude number should not exceed 30% of the critical value (for a single jet - no interference) since as mentioned before an internal hydraulic jump with a depth equal to one-half of the water depth is developed causing limited entrainment. In the case of no jet interference, the critical Froude number is 88 (Equ. 8), (assuming H/D = 20). To minimize the effect of the hydraulic jump, a Froude number of less than 26 (30% F_{cr}) should be maintained if stable near-field is required. The jet behaviour at a Froude number of 30 (Figure 11) shows the jet boundary interferes slightly with the bottom indicating limited entrainment of ambient waters with the jet. An important feature observed in an inclined discharge ($\theta = 20$) is that a small vertical angle of the port's orientation could prevent bottom attachment.

From the foregoing analysis, deep water models may be limited to cases of jets with no interference and with discharge densimetric Froude numbers not exceeding 30% of the critical value. The above limitation is valid for the shallow coastal zone of the Great Lakes.

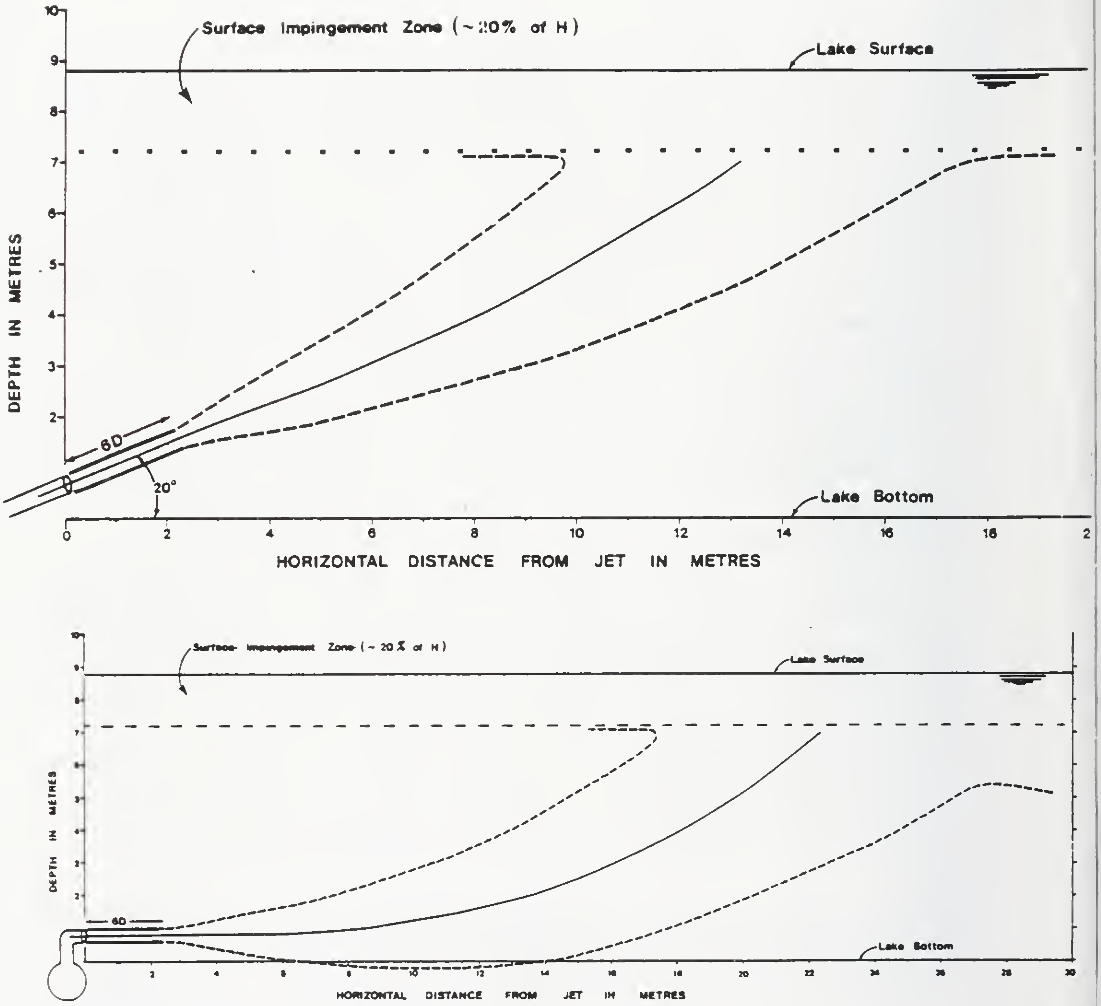

Surface Impingement Zone (~20% of H)
Lake Surface
DEPTH IN METRES
10
9
8
7
6
5
4
3
2
6D
20°
Lake Bottom
0
2
4
6
8
10
12
14
16
18
2
HORIZONTAL DISTANCE FROM JET IN METRES
Surface Impingement Zone (~ 20 % of H)
Lake Surface
DEPTH IN METRES
6D
Lake Bottom
2
4
10
12
14
20
22
30
HORIZONTAL DISTANCE FROM JET IN METRES

FIGURE 11 JET BEHAVIOUR FOR INCLINED AND HORIZONTAL DISCHARGES

Estimates of dilution ratios using deep water models are centre line values and can be determined from the nomograms shown in Figures 12 through 15 for horizontal ($\theta = 0$) and inclined ($\theta = 20^{\circ}$) discharges. These nomograms are similar to those established by Shirazi and Davis (32) for a single jet. The present nomograms, however, are constructed for Froude numbers of up to 50 since as mentioned before, discharges with higher Froude numbers would generally create unstable near-field flow regimes.

To use these nomograms the following steps should be followed:

a) Determine the discharge densimetric Froude number.

b) Calculate the water depth to port diameter (H/D) ratio. To take into account the effect of the surface impingement zone, this ratio should be reduced by 20% (i.e. take 80% of the H/D ratio).

c) From the dilution nomogram, estimate the centre line dilution ratio. To calculate the average dilution, multiply the centreline dilution ratio by 1.35 (10, 21).

d) From the jet width nomogram, determine the jet width.

SHALLOW WATER MODELS

Models for different types of diffuser outfalls in stagnant shallow receiving waters predict average dilution in the near-field taking into consideration re-entrainment effects.

Predictive models for alternating diffusers (with no net horizontal momentum) were developed by using the stratified flow theory (18). The application of the theory assumes that the flow in the centre portion of the diffuser is predominantly two-dimensional where stratified counterflow exists, i.e. surface layer flow moving

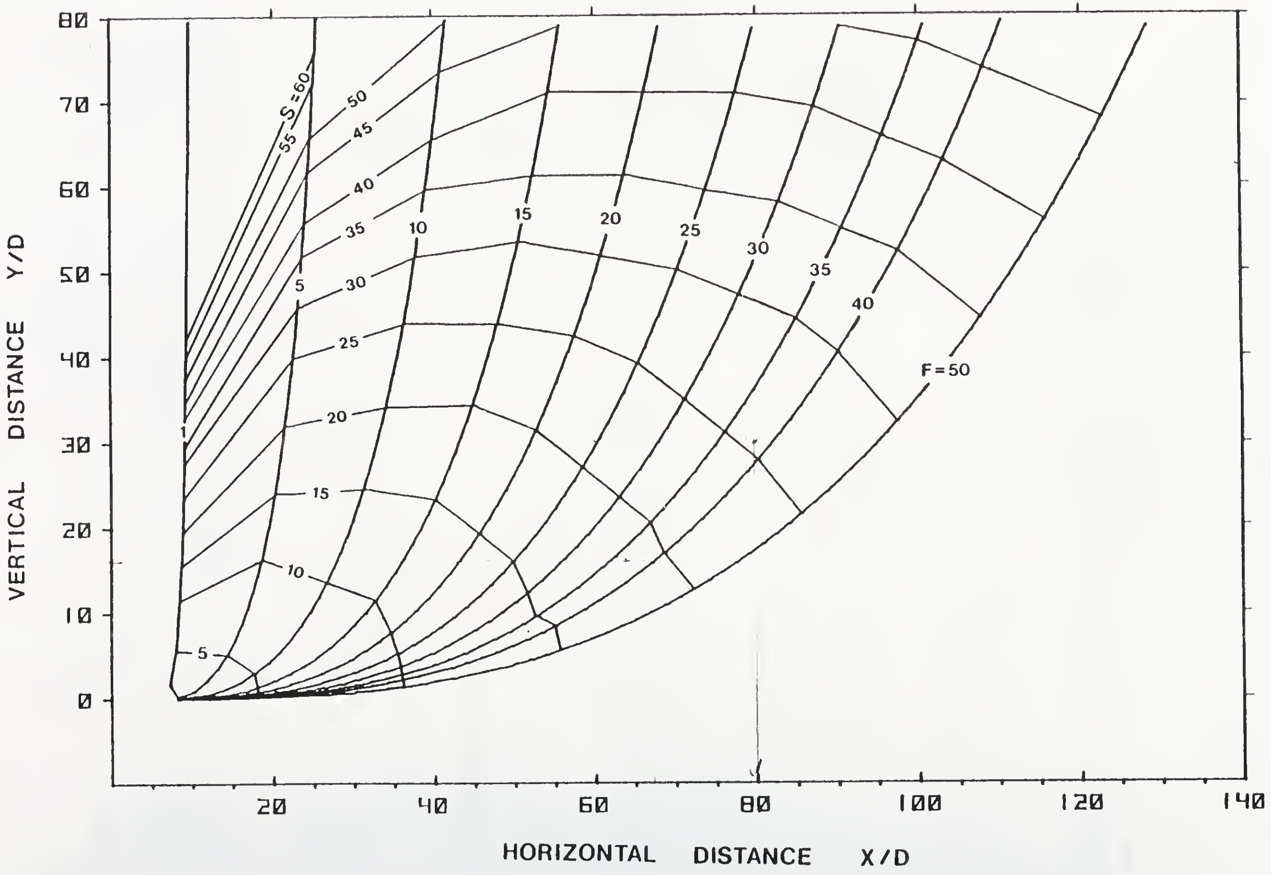


FIGURE 12 DILUTION NO INTERFERENCE $\theta_o = 0°$

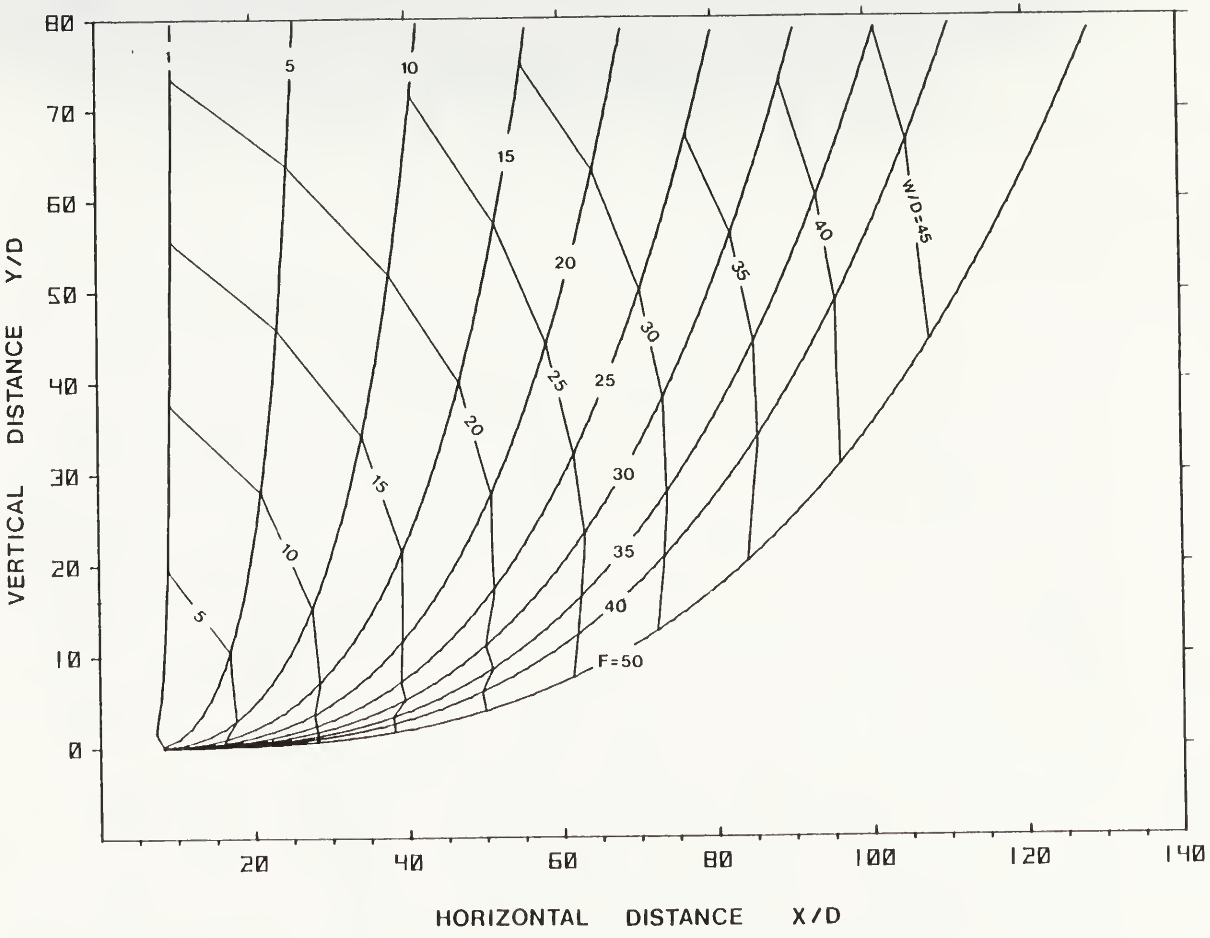


FIGURE 13 JET WIDTH NO INTERFERENCE $\theta_o = 0°$

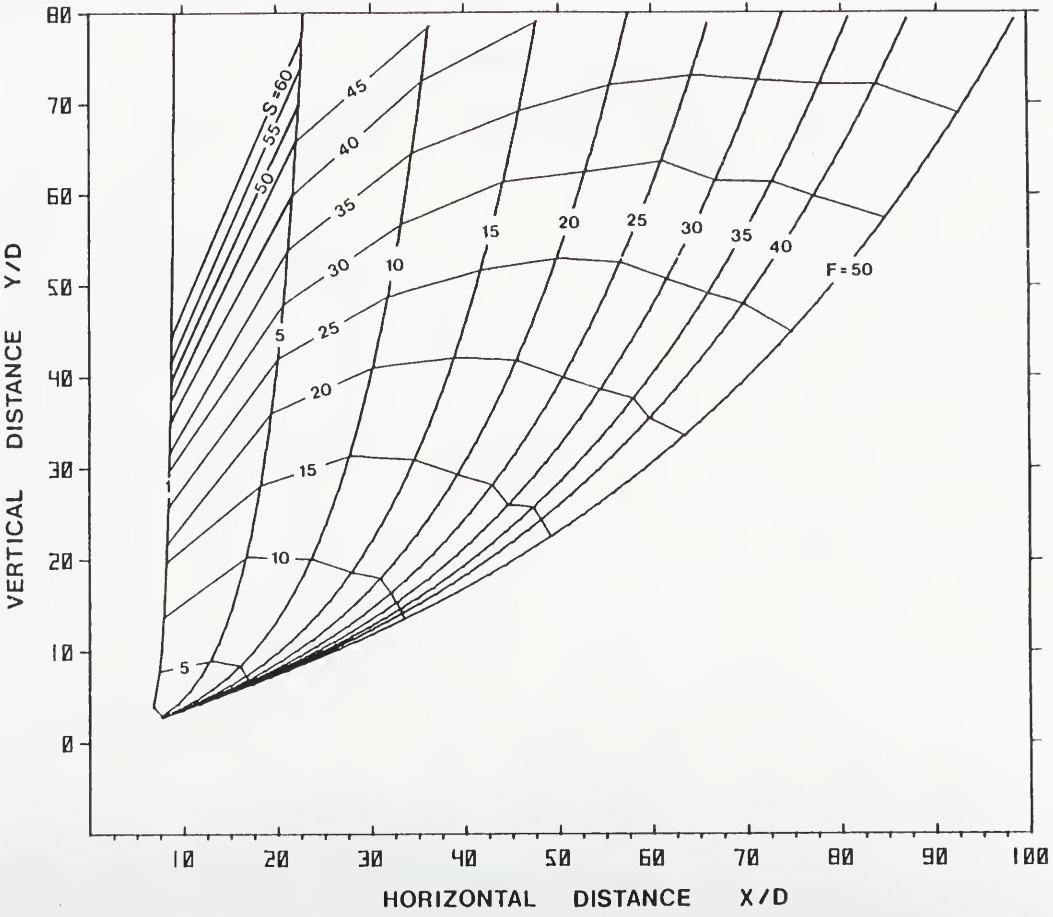


FIGURE 14 DILUTION NO INTERFERENCE $\theta_o = 20°$

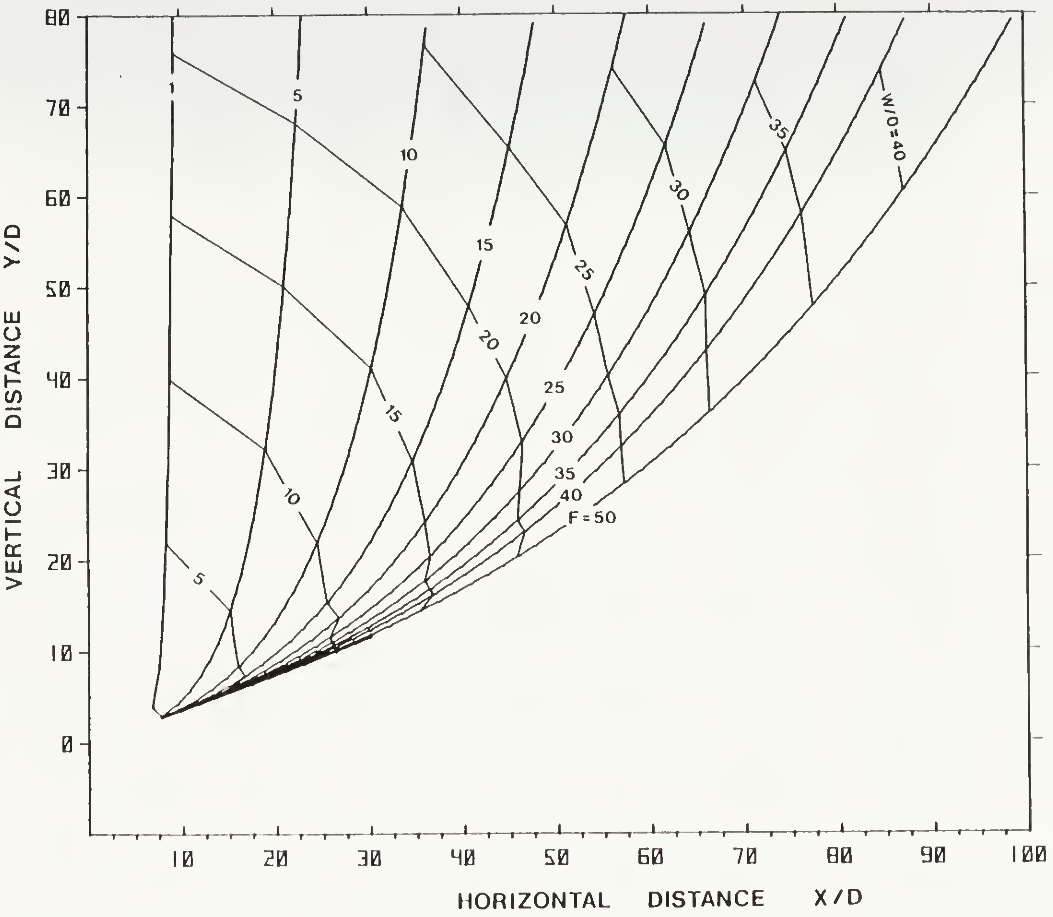


FIGURE 15 JET WIDTH NO INTERFERENCE $\theta_o = 20°$

away from the diffuser and an equal bottom layer flow moving toward the diffuser. Based on this assumption, the maximum average initial dilution can be expressed theoretically (18) in terms of diffuser parameters (ℓ, D):

$$S = c \left(\frac{\ell/D}{F}\right)^{2/3} (H/D) \qquad (10)$$

where c is a coefficient depending on the densimetric Froude number of the exchange flow which is a function of interfacial frictional effects. The value of this coefficient may vary from 0.74 for weak friction to 0.52 for strong frictional effects.

Brooks (10) and Koh and Brooks (21) proposed a similar expression for a centre line dilution for a buoyancy source in deep waters based on an integral analysis. The coefficient (c) in their equation was 0.42 (for an entrainment coefficient of 0.16). This value is in fact equivalent to that of strong frictional effects if one wants to estimate the average dilution (0.42 x 1.35 = 0.57).

The similarity of these findings is expected since the assumption of equal counterflow system used for the above model (Equ. 10) by the latter authors is a characteristic of a deep water diffuser. Jirka and Harleman (18) however, indicated that the average dilution based on equation 10 is the maximum dilution, since experiments conducted for alternating diffusers with ports oriented normal to the diffuser showed successive re-entrainment.

They suggested that in order to use equation 10 for average dilution estimates the horizontal orientation of the ports shoud be controlled by the following theoretical expression:

$$\gamma = \pm \operatorname{Cot}^{-1} \left\{ \frac{1}{\pi} \ln \frac{1 + 2y/L}{1 - 2y/L} \right\} \qquad (11)$$

where: γ is the horizontal angle between the ports and diffuser;
and L is the total diffuser length

The suggested port orientation will result in horizontal angles ranging from 90^0 (normal to the diffuser) to about 0^0 (coinciding with the diffuser) (Figure 16).

To provide conservative estimates of the average initial dilution for an alternating diffuser with ports normal to the diffuser, strong frictional effects may be considered.

Because of the difficulty and complexity of flow phenomenon associated with alternating diffusers in shallow receiving waters, another diffuser "staged diffuser" has been designed recently for coastal regions.

The principal assumption in the staged diffuser design is that the diffuser acts as a continuous source of momentum which produces a well mixed distribution of velocity and concentration in the water column. This momentum should be large to induce an entrainment flow with an induced offshore current to keep waste waters away from the shoreline (7, 8). By using an integral type analysis with the assumption of self-similarity of velocity and concentration profiles the average initial dilution is expressed as:

$$S = 0.61 \left(\frac{\ell H}{D^2}\right)^{1/2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (12)$$

The diffuser length (L) should be equal to or greater than 15 times the water depth, i.e. $L > 15\ H$. For shorter diffusers ($L < 15\ H$), the flow is not fully developed and initial dilutions are somewhat greater than those predicted from the above equation (5).

In equation 12 the discharge densimetric Froude number does not appear since the discharge momentum is so strong that the buoyancy contained in the jet is negligible. The Froude number should be greater than the critical value estimated from equation 6.

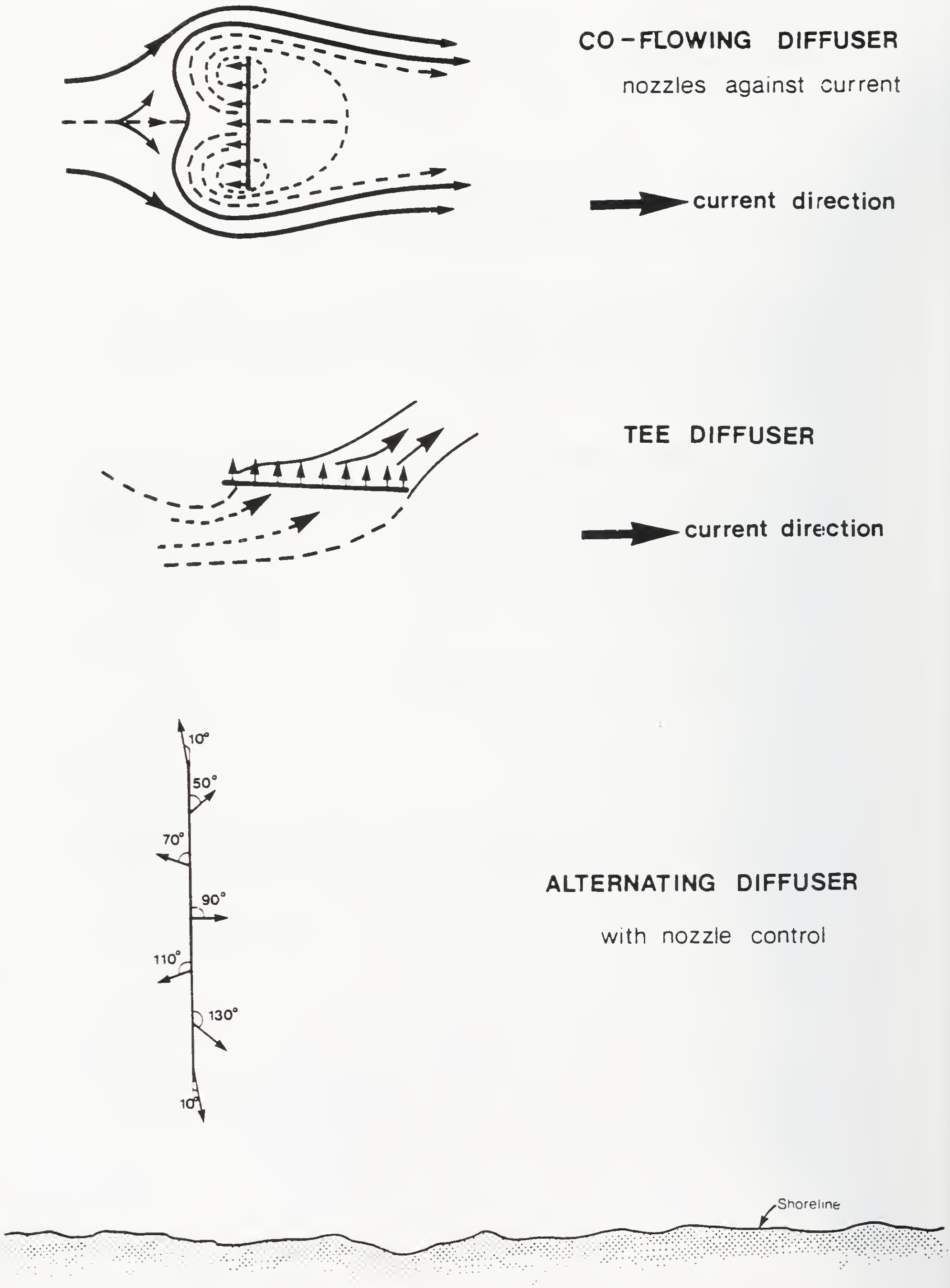


FIGURE 16 FLOW FIELDS ASSOCIATED WITH BASIC DIFFUSER DESIGNS (AFTER ADAMS, 1978)

Another diffuser design which produces a large momentum flux is the unidirectional diffuser. This type of diffuser is called a co-flowing diffuser if the ports are discharging in the direction of the currents. The average initial dilution under stagnant conditions is estimated by Lee et al (23):

$$S = 0.80 \left\{\frac{\ell H}{D^2}\right\}^{1/2} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \qquad (13)$$

This average dilution is higher than that of alternating or staged diffusers indicating that a co-flowing diffuser requires the shortest diffuser length as compared to the alternating or staged diffuser (5). One disadvantage, however, is usually encountered when the currents oppose the diffuser ports resulting in direct re-entrainment (Figure 16). This re-entrainment will cause a sharp reduction in the average dilution and negate the advantages of the current s effect on increasing the dilution.

If a unidirectional diffuser is discharging into perpendicular currents, the diffuser is termed "Tee diffuser". In this case, cross-flow currents will not enchance the average dilution estimated under stagnant conditions due to the interaction among the individual jets (Figure 16) and the dilution will be equal to that of a co-flowing diffuser under stagnant conditions (23). Because of the significant effects of current direction on the unidirectional diffuser performance, this type of diffuser is usually used in rivers.

Initial dilutions for alternating and staged diffusers in shallow receiving waters are easily determined with the aid of the nomograms constructed in Figures 17 and 18 based on equations 10 and 12, respectively.

EXAMPLE

An example of the deep and shallow water models application is illustrated for a typical waste water discharge into the coastal zone of the Great Lakes which would have an average discharge rate

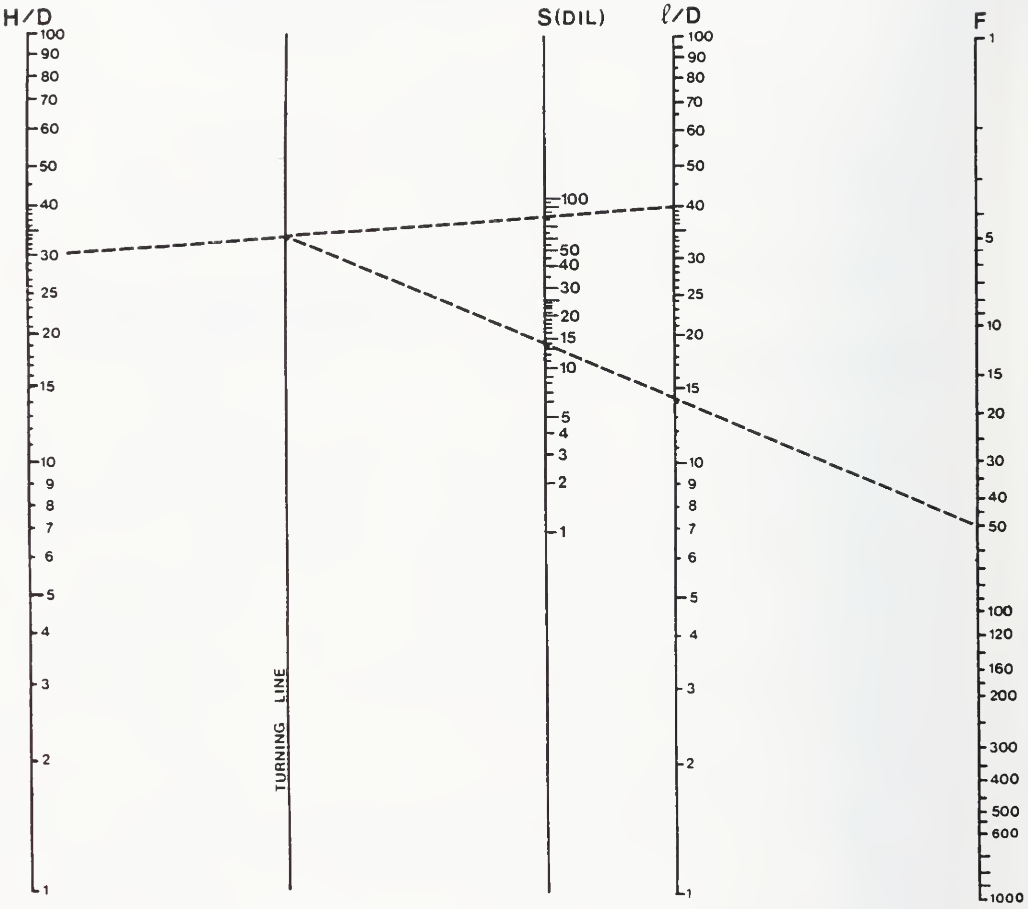


FIGURE 17 DILUTION FACTORS FOR AN ALTERNATING DIFFUSER (STRONG FAR-FIELD FRICTION EFFECTS)

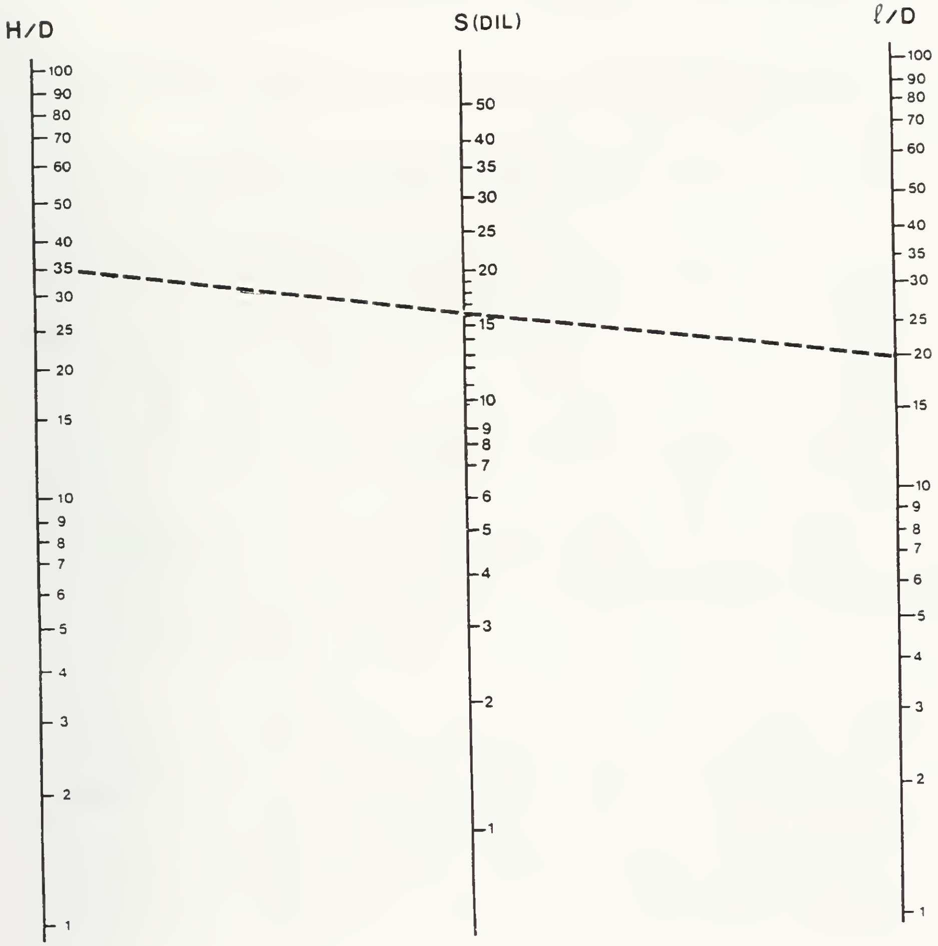


FIGURE 18 DILUTION FACTORS FOR A STAGED DIFFUSER

of 2 m^3/s with a peak rate of 4 m^3/s and an ultimate rate of 7 m^3/s with a peak rate of 14 m^3/s. Estimates of average initial dilution ratios will be attempted for staged and alternating diffusers located in an average water depth of 8 m.

Typical physical variables and governing diffuser parameters are summarized below.

TABLE 2 - Summary of Physical and Governing Diffuser Parameters

Discharge rate (Q) m^3/s	Port Diam. D (m)	Port Spacing ℓ(m) (2ℓ)	# Ports	Jet Vel. U_0 (m/s)	ℓ/D $\frac{2\ell}{D}$	H/D	F
2	0.4	8 or (2ℓ = 16)	25	0.64	20 (or 40)	20	15
4	"	"	"	1.37	"	"	30
7	"	"	"	2.23	"	"	53
14	"	"	"	4.46	"	"	105

If the port's discharge is horizontal (θ = 0), no interference will be expected at discharge rates of 2 and 4 m^3/s and a jet interaction will occur at rates of 7 and 14 m^3/s (Figure 9). For inclined discharges (θ = 20), jet interaction may not occur at all flow rates (Figure 10).

The critical Froude numbers for the case of jet interference are 13 and 36 for alternating and staged diffusers respectively (Equs. 5 & 6). For the case of no jet interference (i.e. single jet) the critical Froude number is 88 (Equ. 8). By comparing the discharge Froude numbers (from Table 2) with the critical values, noting that the critical value for the case of no jet interference must be reduced to 0.30 x 88 = 26.4 at which the depth of the internal hydraulic jump is one-half of the water depth (Froude numbers greater than 26.4, for this example, will produce re-entrainment and interference will occur earlier than that predicted by 'deep' models), the following conclusions are made:

1. A discharge rate of 2 m^3/s at an angle of 0 or 20 is stable with no jet interference; the thickness of the surface impingement zone is about 20% of the water depth.

2. A discharge rate of 4 m^3/s at an angle of 0 or 20 may be considered stable with no jet interference; however, the thickness of the surface impingement zone is 50% of the water depth.

3. Discharge rates of 7 and 14 m^3/s are unstable with jet interference regardless of the jet angle.

The near-field characteristics are summarized in Table 3 together with the suggested models to estimate average initial dilution for each discharge rate. The average initial dilution ratios are shown in Table 4.

TABLE 3 - Near-field Characteristics

Discharge Rate (m^3/s) & Angle (θ)	Near-field Characteristics	Suggested Models
2 (θ = 0 or 20)	Stable, $F = 15$, $F_{cr} = 26$ no jet interference, surface impingement zone is 20% of the water depth	Nomograms, Figs. 12 and 14
4 (θ = 0 or 20)	Stable, $F \simeq F_{cr}$, no jet interference, surface impingement zone is 50% of the water depth	Nomograms, Figs. 12 and 14
7 (θ = 0 or 20)	Unstable, $F = 53$, $F_{cr} = 13$ or 36; jet interference	Nomograms 17 or 18 according to diffuser type
14 (θ = 0 or 20)	Unstable, $F = 105$, $F_{cr} = 13$ or 36; jet interference	Nomogram 17 or 18 according to diffuser type

TABLE 4 - Summary of Average Initial Dilutions

Discharge rate (m^3/s) & (F)	No Jet Interference (stable)*		Jet Interference (unstable)**	
	θ = 0	θ = 20	Alternating	Staged
2 (15)	17	12	-	-
4 (30)	17	10	-	-
7 (52)	-	-	6	12
14 (105)	-	-	3	12

* deep water models (nomograms 12 and 14) - Centreline Dilution x 1.35 = Average dilution

** shallow water models (nomograms 17 and 18)

APPLICATION TO SUBMERGED DIFFUSER OUTFALL DESIGN TO MEET PROVINCIAL WATER QUALITY OBJECTIVES

The total dilution ratios (initial mixing and dispersion) required to reach the acceptable objectives for the following water quality parameters as specified in the Provincial Objectives (29) and Health and Welfare Canada (16) are:

TABLE 5 - Total Dilution Requirement

Water Quality Parameter	Typical Effluent Concentration	Objectives	Total Dilution
Total Kjeldahl Nitrogen (as N)	8 mg/L	0.65 mg/L (a)	21
Total Coliform (with chlorination)	2×10^5 counts/100 ml	1000 counts/100 ml (b)	20
Chlorine residual	0.5 mg/L	.002 mg/L (c)	250

Remarks

a) This objective, is to protect municipal water supply from taste and odour problems. A typical background concentration in Great Lakes water is assumed to be 0.26 mg/L.

b) This objective is to protect beaches during the summer, a typical background concentration of <10 counts/100 ml is assumed.

c) This objective is to protect fish and biota. Background concentration is zero.

The dilution due to dispersion can be estimated according to a simplified model developed by Brooks (9). In Figure 19 dilution ratios at the plume centreline are plotted against a variable

$$\beta \frac{x}{b}, \text{ where } \quad \beta = \frac{12E_0}{Ub} \quad \ldots\ldots \quad (14)$$

in which:
E_0 is the initial dispersion coefficient,
b is the initial plume width which may be taken as the diffuser length,
U is the average steady current velocity, and
x is the longitudinal distance downstream from the diffuser.

In this model, the variation of plume width (w) with distance x is considered linear since experiments carried out in the Great Lakes indicated that this variation does not follow the so called "4/3 Richardson's Power Law" (28). The plume width at any distance x can be expressed as:

$$w = b(1 + \beta x/b) \quad \ldots\ldots \quad (15)$$

Since the plume configuration at any distance x is assumed to be Gaussian, and since the plume width is about 4σ (σ is the standard deviation of the concentration distribution), the concentration at the plume edge can be estimated to be about 0.2 times the plume centreline concentration.

For typical values of $E_0 = 0.1\ m^2/s$ (12), average current velocity (U) = 0.15 m/s, and an initial plume width (same as diffuser length) of 200 m, the dilution due to water movement is about 1:1 at a distance of 1 km from the diffuser.

This result suggests that for non-decaying substances, the downstream concentration reduction in lake plumes parallel to the shore is small. This is consistent with observations reported for several studies in the Great Lakes (28).

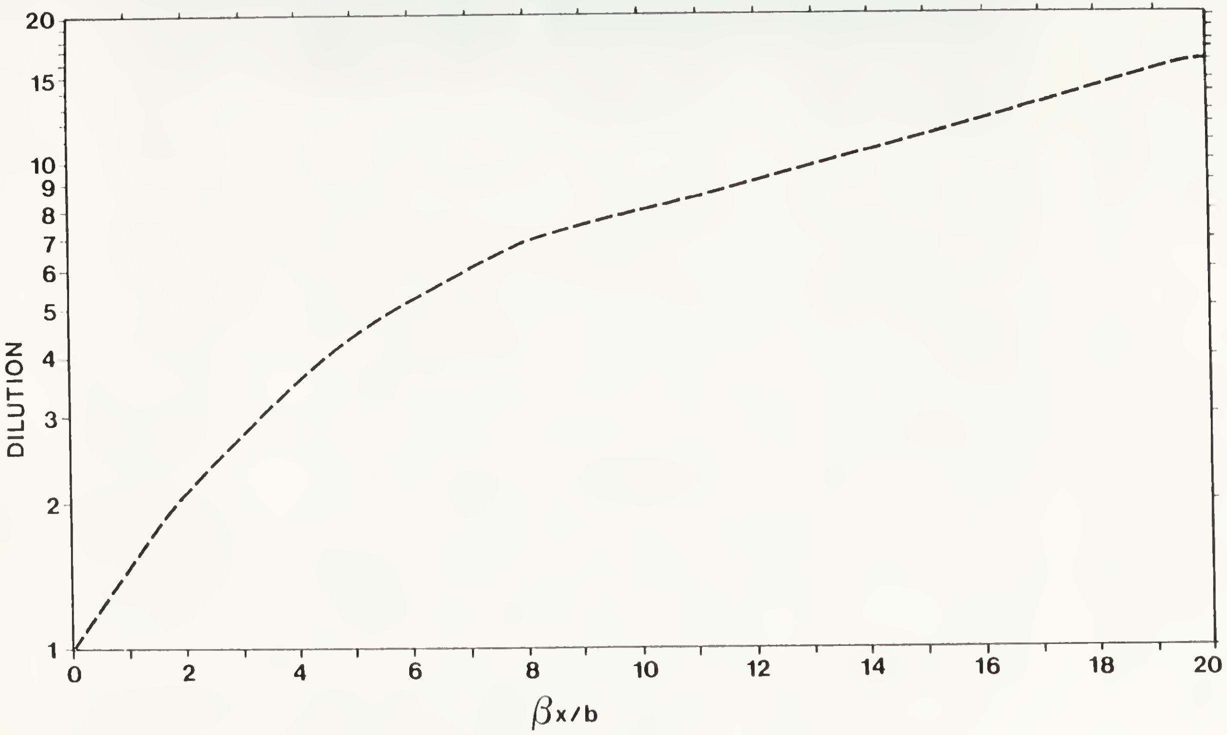


FIGURE 19 DILUTION DUE TO DISPERSION (PLUME CENTRELINE)

The decay due to chemical and biological processes during the winter is expected to be negligible for the water quality parameters shown in Table 5. In general, the dilution due to decay (S_d) can be expressed as:

$$S_d = 1/\mathrm{Exp}\ (-kx/u) = \mathrm{Exp}(kx/u) \quad \text{.............................} \quad (16)$$

where k is the decay rate. For further information on decay rates the reader may refer to Snodgrass (33).

The above model for dispersion provides an order of magnitude estimate of the dilution due to water movement. Transient models such as those developed by Adams et al (4) Koh and Fan (20) and McCorquodale (26) should probably be used to estimate effects of the far-field on the diffuser performance especially when time varying currents cause previously discharged wastes to be re-entrained into the plume.

The initial dilution required should be considered on a case-by-case basis. For example, if a water intake is situated at a distance where the dilution due to water movement and due to the decay process is sufficient to reduce nitrogen and coliform levels, small initial dilution may suffice. Also note the treatment process in water treatment plants will further eliminate these contaminants. For beaches and swimming areas located nearby an outfall, an initial dilution of 20:1 must be achieved. Similarly if effluent chlorination is applied all year-round, a high initial dilution is required to minimize effects of chlorine residual levels on fish and biota. By referring to Table 5, an initial dilution of about 20:1 is a reasonable ratio to be considered for diffuser outfall designs.

For practical purposes this ratio should at least be met for flows up to the average ultimate rates. The ultimate peak rates occur infrequently (2-3 times a year), however, estimates of the magnitude of their impact should be made.

Generally, the near-field flow regime is preferred to be stable to prevent re-entrainment. To obtain a stable near-field together with an average initial dilution of 20:1 (i.e. centreline dilution of about 15:1) a relative water depth at the lower layer of the impingement zone (0.8 H/D) should at least be about 30 or H/D = 40 according to Figure 14. A discharge angle of 20^0 is used to minimize bottom attachment. For a typical average water depth of 8 m, the port diameter will be 0.2 m.

It is also necessary that jet interference should be minimized since as mentioned earlier, the near-field is usually unstable when interference takes place. Furthermore, the discharge Froude number should be less than 30% of the critical value for a single jet to ensure that at least half of the water depth is available for jet entrainment. Since $F_{cr} = 176$ for a single jet (Equ. 8), the discharge Froude number should be much less than 53. From Figure 10, a port spacing to port diameter ratio ($2\ell/D$ for a diffuser with staggered ports) of about 80 or greater should prevent jet interaction for discharges with Froude numbers less than 53 (30% F_{cr}). Roberts (31) suggested that a spacing of ports on one side of the diffuser of two to three times the water depth should prevent jet interaction.

Based on the above, the port spacing (2ℓ) of 20 is recommended and the jet velocity should be less than 1.59 m/s. Assume a velocity of 0.8 m/s (or a Froude number of 26), the port discharge will be .25 m^3/s and the total number of ports required for an average ultimate flow rate of 7 m^3/s is 280 or a diffuser length of 2800 m for $\ell = 10$ m.

For an ultimate peak rate of 14 m^3/s, the jet velocity is 2 m/s and the discharge Froude number is 66 which is somewhat higher than the critical value (53). Deep water models still can be applied, but the effective water depth will be reduced by one-half due to the thickness of the internal hydraulic jump. Thus, for H/D = 20 the average initial dilution will be approximately about 17 (Figure 14). For discharges higher than 14 m^3/s, the near-field will be unstable and shallow water models should be applied.

A high degree of dilution can be obtained by using relatively short diffusers located in shallow waters. The port spacing of these diffusers as a function of the average intial dilution can be estimated using equations 10 and 12 for alternating and staged diffusers:

ℓ (alternating - weak frictional effects) =

$$2 \; S^{3/2} \; g'^{-1/2} \; Q_p/H^{3/2} \qquad (17)$$

ℓ (alternating - strong frictional effects) =

$$3.44 \; S^{3/2} \; g'^{-1/2} \; Q_p/H^{3/2} \qquad (18)$$

$$\ell \text{ (staged diffuser)} = 3.4 \; S^2 \; Q_p/H \; U_o \qquad (19)$$

The jet velocity does not appear in the equation of the alternating diffuser length since the horizontal momentum is zero especially after the jet interference. The use of an alternating diffuser is therefore recommended for low discharge velocities (or low energy head between the treatment facility and the receiving waters). Assume an average water depth of 8 m, and assume strong far-field frictional effects, the port spacing as a function of the port discharge for an alternating diffuser that provides an average initial dilution of 20:1 is: $\ell = 200 \; Q_p$ for $g' = .0045$. The discharge Froude number should be equal to or greater than the critical value (Equ. 5) to satisfy shallow water conditions, or the jet velocity required is:

$$U_o \geq 0.54 g'^{2/3} \; H \; (\ell/Q_p)^{1/3} \qquad (20)$$

Thus, the jet velocity should be equal to or greater than 0.68 m/s (for $H = 8$ m, $g' = .0045$, $\ell/Q_p = 200$). The port spacing (ℓ) should be less than the water depth to ensure interference. The spacing, however, should not be less than one-half of the water depth to ensure uniform concentration along the diffuser, i.e. to minimize "hot spots" (18). Assume $\ell = 5$ m, then $Q_p = 0.025$ m^3/s and the port diameter will be 0.21 m (for a jet velocity of 0.68 m/s). The total number of ports will be 280 with a diffuser length of 1400 m.

In practical situations, the water depth varies along the diffuser length, e.g. 7 m at the nearshore end and 9 m at the offshore end. For the diffuser portion located in 7 m water depth, the port spacing is about 245 Q_p or $Q_p = 0.02$ m^3/s for $\ell = 5$ m, $U_o = 0.64$ m/s and $D = 0.20$. Similarly, the diffuser portion in 9 m water depth will have $Q_p = .03$ m^3/s, $\ell = 5$ m, $U_o = 0.73$ and $D = 0.23$. The total number of ports and their allocation for an average ultimate rate of 7 m^3/s may be suggested as follows:

> For discharges up to 5 m^3/s, 166 ports each of 0.23 m diameter can be arranged in the offshore end at a spacing of 5 m.
>
> For the remaining 2 m^3/s an additional 100 ports, each of 0.20 m diameter at a spacing of 5 m are arranged in the nearshore end.
>
> The total number of ports is 266 with a diffuser length of 1325 m. The average initial dilution is constant for each portion and amount to 20:1

For the ultimate peak rate of 14 m^3/s, the average dilution for this design will be about 15:1.

It should be pointed out that the ports must be controlled according to equation 11 to ensure stratified flow.

Staged diffusers require shorter length than that for alternating diffusers to achieve the same initial diultion. The port spacing as a function of discharge ports, jet velocity and average water depth for a staged diffuser that provides an initial dilution of 20:1 is: $\ell = 1360\ Q_p/U_oH$ (according to equation 19). Note that in staged diffusers, the port spacing should be as small as possible to ensure a uniform source of momentum. For practical purposes, a port spacing equivalent to one-half of the water depth may be used. Assume $\ell = 4$, $H = 8$ m, then, $U_o = 42.5\ Q_p$ or $D = 0.17$ m $(Q_p = \pi D^2/4U_o)$.

The discharge Froude number should be greater than the critical value (equation 6) or:

$$U_o > 1.79H^{3/4}\ \ell^{1/4}D^{-1/2}g'^{1/2} \quad \text{.................................} \quad (21)$$

For g' = .0045, the discharge velocity should be equal to or greater than 1.95 m/s or Q_p should be equal to or greater than 0.044 m^3/s. The advantage of using a staged diffuser is realized when the jet velocity is high enough to reduce the diffuser length, provided that the available energy head is sufficient for these velocities. If the port discharge is taken to be double the above rate, i.e. 0.088 m^3/s, the number of ports will be 80 for a total discharge rate of 7 m^3/s and the diffuser length will be 300 m. The average initial dilution for this design will be constant (20:1) for all the flow rates. In the case of an offshore bottom slope the dilution is improved by a factor of $(1 + \Delta H/2H)^{1/2}$ where ΔH is the increase in depth along the diffuser length (24).

SUMMARY AND CONCLUSIONS

Predictions of initial dilution ratios under stagnant conditions based on 'deep' and 'shallow' water models are presented in terms of multiport diffuser design parameters: port spacing (ℓ), port diameter (D) and jet velocity (U_o). Diffusers designed according to the theory of deep water models is based on a stable near-field which is not prone to horizontal circulation and re-entrainment.

The interference of individual jets should be minimized and the discharge Froude number should not exceed 30% of the critical value to ensure that the depth of the internal hydraulic jump is not exceeding one-half of the water depth. The spacing of ports on one side of the diffuser should be at least 2.5 times the water depth.

Diffusers based on deep water models to achieve a certain average initial dilution are longer than those based on shallow water models. The criteria for shallow water conditions should be met according to diffuser type provided that jet interaction occurs. The mechanics of diffusers in shallow waters can be classified according to the diffuser type.

1. Alternating Diffusers

These diffusers provide a buoyance source since the net horizontal momentum (after jet interference) is negligible. The Froude number necessary to produce an unstable near-field is usually small and therefore these diffusers are used for low energy head between the treatment facility and the receiving water. The discharge ports should be controlled to establish a stratified flow in the far-field thus minimizing the degree of circulation and re-entrainment in the near-field. The port spacing (ℓ) should be less than the water depth. To prevent isolated "hot spots" the spacing (ℓ) is recommended to be larger than one-half of the water depth.

2. Staged Diffusers

This type of diffuser acts as a momentum line source. The advantage of using staged diffusers is realized when the discharge velocity is high enough to reduce the diffuser length. The length, however, should not be less than 15 times the water depth. Staged diffusers are recommended when the available energy head is sufficient to sustain high velocities.

The initial dilution required should be considered for a case-by-case basis depending on water uses in the area. An average dilution of 20:1, however, may be a reasonable ratio to be considered in the design of outfalls discharging into the Great Lakes. By achieving an initial dilution of 20:1, the provincial objectives for coliform counts and nitrogen compounds will be met immediately.

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